ÖVNINGSEXEMPEL

BERÄKNA $F(s)$ OM

a) $f(t) = E$

b) $f(t) = E \cdot e^{-at}$

c) $f(t) = \sin(bt)$

d) $f(t) = \cos(bt)$

a) $$F(s) = \int_0^\infty E \cdot e^{-st}\,dt = \left[-\frac{E}{s}\cdot e^{-st}\right]_0^\infty = \frac{E}{s}$$

b) $$F(s) = \int_0^\infty E \cdot e^{-at} \cdot e^{-st}\,dt =$$

$$= E \cdot \int_0^\infty e^{-(s+a)t}\,dt =$$

$$= E \cdot \left[-\frac{1}{s+a} \cdot e^{-(s+a)t}\right]_0^\infty = \frac{E}{s+a}$$

c) & d)

BETRAKTA $f(t) = e^{jbt}$

$$F(s) = \int_0^\infty e^{jbt} \cdot e^{-st} dt$$

$\mathcal{L}(\sin(bt)) = \operatorname{Im} F(s)$ OCH

$\mathcal{L}(\cos(bt)) = \operatorname{Re} F(s)$ TY

$$e^{jbt} = \cos(bt) + j\sin(bt)$$

$$F(s) = \int_0^\infty e^{-(s-jb)t} dt =$$

$$= \left[-\frac{1}{s-jb} \cdot e^{-(s-jb)t} \right]_0^\infty =$$

$$= \frac{1}{s-jb} = \frac{s+jb}{s^2+b^2} = \underbrace{\frac{s}{s^2+b^2}}_{\operatorname{Re} F(s)} + j \cdot \underbrace{\frac{b}{s^2+b^2}}_{\operatorname{Im} F(s)}$$

$$\mathcal{L}(\sin(bt)) = \frac{b}{s^2+b^2}$$

$$\mathcal{L}(\cos(bt)) = \frac{s}{s^2+b^2}$$